5.3: The Other Trigonometric Functions

If (x, y) is the terminal point for for angle t.

- Tangent of angle t: $\boxed{\tan(t) = \frac{y}{x}}$ for $x \neq 0$. Also $\tan(t) = \frac{\sin(t)}{\cos(t)}$ when $\cos(t) \neq 0$.
- Cotangent of angle t: $\boxed{\cot(t) = \frac{x}{y}}$ for $y \neq 0$. Also $\cot(t) = \frac{\cos(t)}{\sin(t)}$ when $\sin(t) \neq 0$.
- Secant of angle t: $\boxed{\sec(t) = \frac{1}{x}}$ for $x \neq 0$. Also $\sec(t) = \frac{1}{\cos(t)}$ when $\cos(t) \neq 0$.
- Cosecant of angle t: $\boxed{\csc(t) = \frac{1}{y}}$ for $y \neq 0$. Also $\csc(t) = \frac{1}{\sin(t)}$ when $\sin(t) \neq 0$.
- Reciprocal identity for Tangent: $\tan(t) = \dfrac{1}{\cot(t)}$ when $\cot(t) \neq 0$.
- Reciprocal identity for Cotangent: $\cot(t) = \dfrac{1}{\tan(t)}$ when $\tan(t) \neq 0$.

Now, you can complete the left table in Question 1.

- **Finding Trigonometric Functions of θ**
  - Find the reference angle, t, for θ.
  - Use the table of sine/cosine values for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ to find the **absolute value** of the sine and cosine.
  - Subtract/add enough 2π to the angle to find the coterminal angle and its **quadrant**.
  - Find the **sign** of the sine and cosine in the correct **quadrant**.
  - Use the formula for $\tan, \cot, \sec$ and $\csc$ to find the other values.
  - That is, $\sin(\theta) = \pm(\sin(t))$ and $\cos(\theta) = \pm(\cos(t))$

Now, you can complete the right table in Question 1.

- **Periodic functions:** A function is periodic if there is a positive number p such that $f(t) = f(t+p)$. That is, the function repeats itself after time p has passed. We call p the period if it is the **smallest** such number.
- **Periodic properties of Sine and Cosine**:

  The function $\sin(t)$ and $\cos(t)$ have period 2π that is $\sin(t) = \sin(t + 2\pi)$ and $\cos t = \cos(t + 2\pi)$ for all t. We observed that the value of sine and cosine on the circle repeats itself after $2k\pi$ for k an integer.
- **Optimization with Trigonometric functions:** Maximum/minimum value of $\sin(t)$ and $\cos(t)$ are $1/-1$. To find Maximum/minimum of a transformation of sine and cosine function, solve an inequality. *Now, you can complete Questions 2 and 3.*

Summary of identities

- **Pythagorean identity:**

$$\sin^2(t)+\cos^2(t)=1 \qquad \text{(Main equation)}$$

$$\sin^2(t)=1-\cos^2(t) \qquad \text{(Obvious alternative)}$$

$$\cos^2(t)=1-\sin^2(t) \qquad \text{(Obvious alternative)}$$

$$\sec^2(t)=\tan^2(t)+1 \qquad \left(\text{A derivation: } \underbrace{\frac{\sin^2(t)}{\cos^2(t)}}_{\tan^2(t)}+\underbrace{\frac{\cos^2(t)}{\cos^2(t)}}_{1}=\underbrace{\frac{1}{\cos^2(t)}}_{\sec^2(t)}\right)$$

$$\csc^2(t)=1+\cot^2(t) \qquad \left(\text{Another derivation: } \underbrace{\frac{\sin^2(t)}{\sin^2(t)}}_{1}+\underbrace{\frac{\cos^2(t)}{\sin^2(t)}}_{\cot^2(t)}=\underbrace{\frac{1}{\sin^2(t)}}_{\csc^2(t)}\right)$$

- **Even and Odd**:

Odd:

$\sin(-t)=-\sin(t)$

$\tan(-t)=-\tan(t)$

$\cot(-t)=-\cot(t)$

$\csc(-t)=-\csc(t)$

Even:

$\cos(-t)=\cos(t)$

$\sec(-t)=\sec(t)$

We use these identities to compute different trigonometric values.

1. Complete the table for all trig functions.

t	$\sin(t)$	$\cos(t)$	$\tan(t)$	$\cot(t)$	$\sec(t)$	$\csc(t)$
0				DNE		DNE
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$			DNE		DNE	

t	$\sin(t)$	$\cos(t)$	$\tan(t)$	$\cot(t)$	$\sec(t)$	$\csc(t)$
$\frac{3\pi}{2}$			DNE		DNE	
$\frac{5\pi}{3}$						
$\frac{7\pi}{6}$						
$\frac{7\pi}{4}$						
π				DNE		DNE

2. **Biology (the Predator Prey Model):** In many models of population with predator and prey when the population of prey starting to increase, the population of predator increases. After a while, the increase in population of predator causes the population of the prey after a while causes the population of prey to decrease. And the decrease in population of prey causes the decrease in population of predator. This is a cycle that repeats itself and can be modeled by a simple periodic functions such as sine and cosine.

   Let $N(t) = 1200\sin(3t) + 2500$ be the population of prey over time. Find the maximum population and length of time between successive periods of maximum population.

3. **Biology (Blood Pressure):** The equation $P(t) = 20\sin(2\pi t) + 100$ models the blood pressure for a healthy 20-year old, P, where t represents time in seconds. **(a)** Find the blood pressure after 15 seconds. **(b)** What are the maximum and minimum blood pressures?